CSE 2105
Digital Logic Design

NUMBER SYSTEM

Number

- A number is a mathematical value used for counting or measuring or labelling objects.
- Numbers are used to perform arithmetic calculations.

Number Systems

A number system is defined as a system of writing to express numbers.

It provides a unique representation of every number and allows us to operate arithmetic operations like addition, subtraction, multiplication and division.

Base: total number of digits in the number system.

Types of number systems

There are various types of number systems in mathematics. The four most common number system types are:

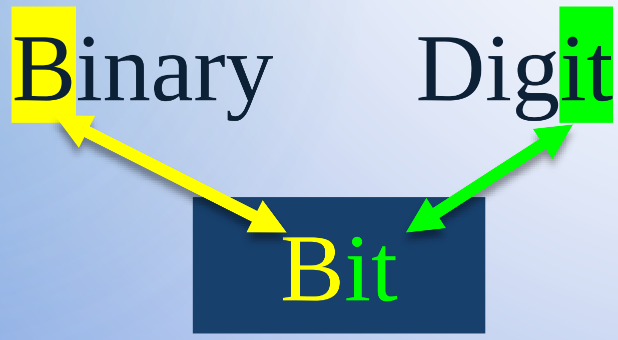
Number system	Details
Decimal number system	• Base- 10 • Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Binary number system	• Base- 2 • Digits: 0, 1
Octal number system	• Base- 8 • Digits: 0, 1, 2, 3, 4, 5, 6, 7
Hexadecimal number system	• Base- 16 • Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Decimal Number System (Base 10 Number System)

- The decimal number system has a base of 10 because it uses ten digits from 0 to 9.
- The maximum value of a single digit is 9. Each position of a digit represents a specific power of the base (10). We use this number system in our day-to-day life.
- Decimals play a vital role in mathematics, especially in the branches like number systems, measurement, geometry etc. To find the area and volume of the objects, we use decimals.

Binary Number System (Base 2 Number System)

- The base 2 number system is also known as the Binary number system wherein, only two binary digits exist, i.e., 0 and 1.
- Bit stands for binary digit. A bit in computer terminology means either a 0 or a 1.
- A binary number consisting of n bits is called an n-bit number.



Octal Number System (Base 8 Number System)

- In the octal number system, the base is 8 and it uses numbers from 0 to 7 to represent numbers.
- Octal numbers are commonly used in computer applications.
- All the modern generation computing system uses 16-bit, 32-bit or 64-bit word which is further divided into 8-bit words.
- Also in the aviation sector, the octal numbers are used in the form of code.

Hexadecimal Number System (Base 16 Number System)

- Hexa-decimal number system has total 16 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). The symbols A, B, C, D, E and F represent the decimal values 10, 11, 12, 13, 14 and 15 respectively.
- Hexadecimal Number System is commonly used in Computer programming and Microprocessors.
- It is also used to describe locations in memory for every byte.

Converting a Number of Another Base to a Decimal Number

At first we need to know how a number (specially decimal number) develops from positional system.

Lets assume a number 4892 (Four thousand Eight hundred Ninety Two).

There are four digits 4, 8, 9, 2.

Digit	**4**	**8**	**9**	**2**
Position	3	2	1	0
Positional weight	thousand	hundred	ten	unity
	$1000 \ or \ 10^3$	$100 \ or \ 10^2$	$10 \ or \ 10^1$	$1 \ or \ 10^0$
Positional Value	4000 Four Thousand	800 Eight Hundred	90 Ninety	2 Two

So we can represent the number as

$$(4892)_{10} = (4000 + 800 + 90 + 2)_{10} = (4 \times 10^3 + 8 \times 10^2 + 9 \times 10^1 + 2 \times 10^0)_{10}$$

Converting a Number of Another Base to a Decimal Number

We can further extend this concept for fraction number.

Lets assume a number 4892.653

Digit	**4**	**8**	**9**	**2**	**.**	**6**	**5**	**3**
Position	3	2	1	0	Radix point	-1	-2	-3
Positional weight	1000 $or\ 10^3$	100 $or\ 10^2$	10 $or\ 10^1$	1 $or\ 10^0$		0.1 or $\frac{1}{10}\ or\ 10^{-1}$	0.01 or $\frac{1}{100}\ or\ 10^{-2}$	0.001 or $\frac{1}{1000}\ or 10^{-3}$
Positional Value	4000	800	90	2	.	$\frac{6}{10}$ or 0.6	$\frac{5}{100}$ or 0.06	$\frac{3}{1000}$ or 0.003

So we can represent the number as

$$(4892.653)_{10} = (4000 + 800 + 90 + 2 + 0.6 + 0.05 + 0.003)_{10}$$
$$= (4 \times 10^3 + 8 \times 10^2 + 9 \times 10^1 + 2 \times 10^0 + 6 \times 10^{-1} + 5 \times 10^{-2} + 3 \times 10^{-3})_{10}$$

Converting a Number of Another Base to a Decimal Number

So we can summarize the conversion of a number of any base to decimal into two steps.

- **Step 1:** Multiply each digit by the base of that number system to the power of the position value of that digit.
- **Step 2:** Calculate the sum of these products.

Example 1 (Binary to Decimal): $(11011.1001)_2 = (?)_{10}$

$$(11011.1001)_2 = (1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4})_{10}$$
$$= (16 + 8 + 0 + 2 + 1 + 0.5 + 0 + 0 + 0.0625)_{10} = (27.5625)_{10}$$

Example 2 (Octal to Decimal): $(4706.25)_8 = (?)_{10}$

$$(4706.25)_8 = (4 \times 8^3 + 7 \times 8^2 + 0 \times 8^1 + 6 \times 8^0 + 2 \times 8^{-1} + 5 \times 8^{-2})_{10}$$
$$= (2048 + 448 + 0 + 6 + 0.25 + 0.078125)_{10} = (2502.328125)_{10}$$

Example 3 (Hexadecimal to Decimal): $(1AF.2B)_{16} = (?)_{10}$

$$(1AF.B2)_{16} = (1 \times 16^2 + 10 \times 16^1 + 15 \times 16^0 + 11 \times 16^{-1} + 2 \times 16^{-2})_{10}$$
$$= (256 + 160 + 15 + 0.6875 + 0.0078125)_{10} = (431.6953125)_{10}$$

Converting a Decimal Number to a Number of Another Base

At first we need to know how the digits of a number (specially decimal number) can be extracted from its positional weight.

Lets assume a number 4892 (Four thousand Eight hundred Ninety Two).

There are four digits 4, 8, 9, 2. We need to extract these digits from the number

Digit	**4**	**8**	**9**	**2**
Position	3	2	1	0
Positional weight	thousand	hundred	ten	unity
	$1000\ or\ 10^3$	$100\ or\ 10^2$	$10\ or\ 10^1$	$1\ or\ 10^0$
Positional Value	4000 Four Thousand	800 Eight Hundred	90 Ninety	2 Two

- If we divide 4892 by 10, then the dividend is 489 and the remainder is 2.
- Then if we divide the previous dividend 489 by 10, then the dividend is 48 and the remainder is 9.
- If we further divide the previous dividend 48 by 10, then the dividend is 4 and the remainder is 8.
- Similarly if we divide the previous dividend 4 by 10, then the dividend is and the remainder is 4.

This method is called Division-Remainder Method.

Converting a Decimal Number to a Number of Another Base

But thing goes different after radix point!!!

Lets assume a number 4892.653.

We have already learned to extract the digits before radix point. Now we need to extract these digits from the number after radix point.

Digit	4	8	9	2	.	6	5	3
Position	3	2	1	0	Radix point	-1	-2	-3
Positional weight	1000 $or\ 10^3$	100 $or\ 10^2$	10 $or\ 10^1$	1 $or\ 10^0$		$0.1\ or\ \frac{1}{10}\ or\ 10^{-1}$	$0.01\ or\ \frac{1}{100}\ or\ 10^{-2}$	$0.001\ or\ \frac{1}{1000}\ or 10^{-3}$
Positional Value	4000	800	90	2	.	$\frac{6}{10}$ or 0.6	$\frac{5}{100}$ or 0.06	$\frac{3}{1000}$ or 0.003

- If we multiply 0.623 by 10, then the product is 6.23 where the integer value is 6 and the fraction is 0.23
- Then if we multiply the fraction part of previous product 0.23 by 10, then the product is 2.3 where the integer value is 2 and the fraction is 0.3
- If we multiply the fraction part of previous product 0.3 by 10, then the product is 3 where the integer value is 3 and the fraction is 0.

Converting a Decimal Number to a Number of Another Base

So we can summarize the conversion of a decimal number to a number of another base into two sections

Section 1 (Before the radix point):

- **Step 1:** Divide the integer part of the decimal number to be converted by the base of the new number system.
- **Step 2:** Record the remainder from Step 1 on the right.
- **Step 3:** Divide the quotient of the previous divide by the base again and repeat step 2.

Section 2 (After the radix point):

- **Step 1:** Multiply the fraction part of the decimal number to be converted by the base of the new number system.
- **Step 2:** Record the integer part from Step 1 on the left.
- **Step 3:** Multiply the fraction part of the previous product by the base again and repeat step 2.

Converting a Decimal Number to a Number of Another Base

- Convert the Decimal number $(25.12)_{10}$ into a Binary number.
- Solution:

		Remainder
2	25	
2	12	1
2	6	0
2	3	0
2	1	1
	0	1

Integer	.12
	× 2
0	.24
	× 2
0	.48
	× 2
0	.96
	× 2
1	.92

Hence, $(25.12)_{10} = (11001.0001)_2$

Converting a Decimal Number to a Number of Another Base

- Convert the Decimal number $(952)_{10}$ into an Octal number.
- Solution:

Example

$952_{10} = ?_8$

Solution:

8	952	Remainders
	119	0
	14	7
	1	6
	0	1

Hence, $952_{10} = 1670_8$

Hence, $(952)_{10} = (1670)_8$

Converting a Decimal Number to a Number of Another Base

- Convert the Decimal number $(455.72)_{10}$ into a Hexa-Decimal number.
- Solution:

16	459	Remainder
16	28	11(B)
16	1	12(C)
	0	1

Integer	
	.72 ×16
11(B)	.52 ×16
8	.32 × 2
5	.12 × 16
1	.92

Hence, $(455.72)_{10}$= $(1CB.B851)_{16}$

Converting a Binary Number to its Equivalent Octal Number

- **Method**
- **Step 1:** Divide the digits into groups of three starting from the right.
- **Step 2:** Convert each group of three binary digits to one octal digit using the method of binary to octal conversion.

Converting a Binary Number to its Equivalent Octal Number

- **Convert the Binary number $(1101010)_2$ into an Octal number.**
- **Solution:**

001	101	010
1	5	2

- Hence, $(1101010)_2 = (152)_8$

Converting a Binary Number to its Equivalent Hexa-decimal Number

- **Method**
- **Step 1:** Divide the digits into groups of four bits starting from the right.
- **Step 2:** Convert each group of four binary digits to one hexa-decimal digit using the method of binary to hexa-decimal conversion.

Converting a Binary Number to its Equivalent Hexa-decimal Number

- **Convert the Binary number $(110101010.00111)_2$ into a Hexa-decimal number.**
- **Solution:**

0110	1010	1010	.	0011	1000
6	10(A)	10(A)		3	8

- Hence, $(110101010.00111)_2 = (6AA.38)_8$

Converting an Octal number to its Equivalent Binary number

- **Method**
- Convert each digit of the Octal number to a 3 digit binary number.
- **Example:** $(357.14)_8 = (?)_2$
- **Solution:** 3 = 011

  5 = 101

  7= 111

  1= 001

  4 = 100
- Hence, $(357.14)_8 = (011101111.001100)_2$

Converting a Hexadecimal number to its Equivalent Binary number

- **Method**
- Convert the decimal equivalent of each hexadecimal digit to a 4 digit binary number.
- **Example:** $(2AB.C4)_{16} = (?)_2$
- **Solution:** 2 = 2 = 0010

  A = 10 = 1010

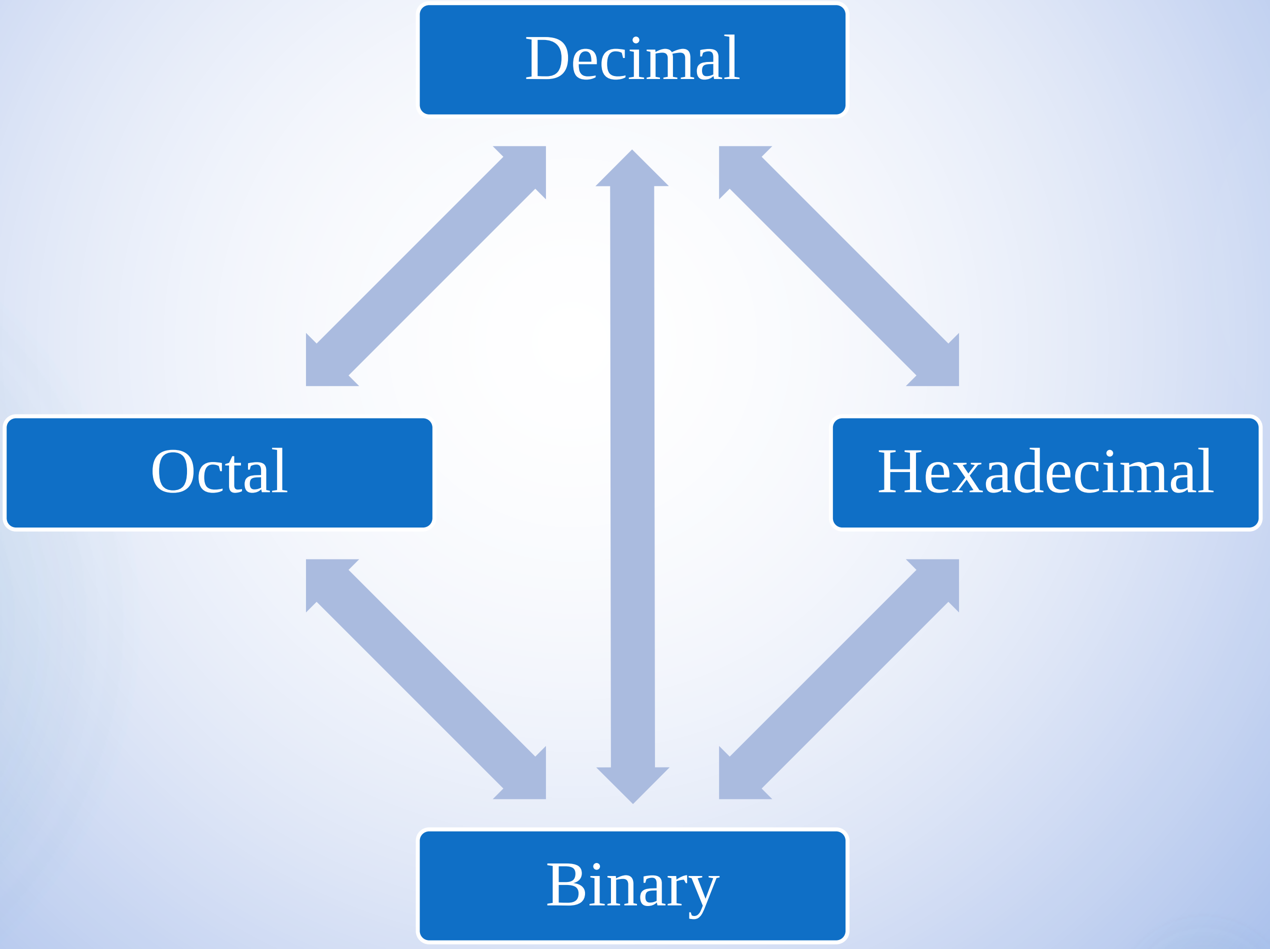
  B = 11 = 1011

  C = 12 = 1010

  4 = 4 = 1011

Hence, $(2AB.C4)_{16} = (001010101011.10101011)_2$

Number System Base Conversion



Binary Addition

- Binary Addition rules
- 0 + 0 = 0
- 1 + 0 = 1
- 0 + 1 = 1
- 1 + 1 = 0 (Carry 1)

Binary Addition

- **Example 1:** $(10001.001)_2 + (11101.010)_2 = ?$
- **Solution:**
- **1**
- 1 0 0 0 1. 0 0 1
- (+)1 1 1 0 1. 0 1 0
- ____________________
- 1 0 1 1 1 0. 0 1 1

Binary Subtraction

- Binary Subtraction rules
- 0 – 0 = 0
- 1 – 0 = 1
- 1 – 1 = 0
- 0 – 1 = 1 (Borrow 1)

Binary Subtraction

- **Example 1:** (10001.100) – (11101.010)
- **Solution:**
- **1**
- 1 0 0 0 1. 1 0 0
- (−)1 1 1 0 1. 0 1 0
- ______________________
- 1 0 1 1 1 0. 0 1 1